Theory and Applications of Logarithmic Amplifiers

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A number of instrumentation applications can benefit from the use of logarithmic or exponential signal processing techniques. The design and use of logarithmic/exponential circuits are often associated with involved temperature compensation requirements and difficult to stabilize feedback loops. For these considerations and others, designers tend to avoid these circuits. Hybrid and modular logarithmic/exponential devices are available commercially, but are quite expensive and earn very high profits for their manufacturers.

The theory and construction of these circuits are actually readily understood. *Figure 1* shows an amplifier which provides a logarithmic output for a linear input current or voltage. For input currents, the circuit will maintain 1% logarithmic conformity over almost 6 decades of operation. This circuit is based, as are most logarithmic circuits, on the inherent logarithmic relationship between collector current and V_{BE} in bipolar transistors. Q1A functions as the logging transistor in this circuit and is enclosed within A1A's feedback loop, which includes the 15.7 kΩ-1 kΩ divider. The circuit's input will force A1A's output to achieve whatever value is required to maintain its summing junction at zero potential. Because Q1A's response is dictated by the logarithmic relationship between collector current and V_{BE}, the output of A1A will be the logarithm of the circuit input. A1B and Q1B provide compensation for Q1A's V_{BE} temperature dependence. A1B servos Q1B's collector current to equal the 10 µA current established by the LM329 reference diode and the 700 kΩ resistor. Since Q1B's collector current cannot vary, its V_{BE} is also fixed. Under these conditions only Q1A's V_{BE} will be affected by the circuit's input. The circuit's output is a function of:

$$E_{OUT} = \frac{15.7k + 1k}{1k} (V_{BE}Q1B - V_{BE}Q1A)$$

For Q1A and Q1B operating at different collector currents, the V_{BE} difference is:

$$\Delta V_{BE} = \frac{KT}{q} \log_e \frac{I_{CQ1A}}{I_{CQ1B}}$$

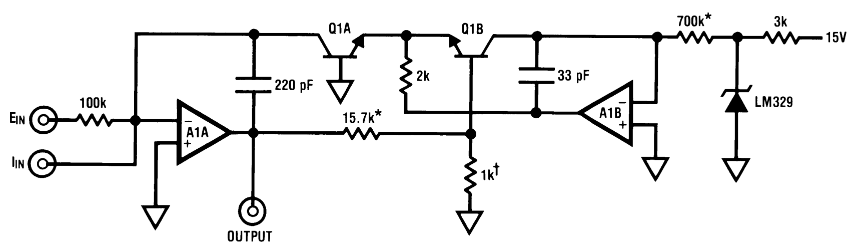
where K=Boltzmann's constant
T=temperature ˚K
q=charge of an electron.

If both equations are combined, the circuit output for a voltage input is:

$$E_{OUT} = \frac{-KT}{q} \frac{15.7k + 1k}{1k} \log_e \frac{E_{IN} \bullet 700k}{6.9V \bullet 100k}$$

where $6.9V = V_Z$ of LM329
100k=input resistor
$E_{IN} \geq 0$.

This confirms that the circuit output voltage is logarithmically related to the circuit's input. Without some form of compensation, the scale factor will change with temperature. The simplest way to avoid this is to have the 1 kΩ value vary with temperature. For the device shown, compensation is within 1% over −25˚C to +100˚C. The circuit's gain is set by the 15.7 kΩ-1 kΩ divider to a factor of 1V/decade.



*1% film resistor
†1 kΩ (±1%) at 25˚C, +3500 ppm/˚C.
Available from Vishay Ultronix,
Grand Junction, CO, Q81 Series.
A1A, A1B = LF412 dual
Q1A, Q1B = LM394 dual

FIGURE 1.

This circuit may be easily turned around to generate exponentials. In *Figure 2*, Q1A is driven from the input via the 15.7 kΩ divider. Q1B's collector current varies exponentially with its V_{BE}, and A1B provides a voltage output representation of this action.

These circuits are easy to construct and use if a few considerations are kept in mind. Because of the V_{BE} and scale factor temperature dependences, it is important that Q1A, Q1B and the 1 kΩ resistor be kept at the same temperature. Since Q1 is a dual monolithic device, both halves will track. The resistor should be mounted as closely as possible to Q1, and these components should be kept away from air currents or drafts. The KT/q factor for which the resistor compensates varies at about 0.3%/˚C, so a few degrees difference between Q1 and the resistor will introduce significant error.

Once the theory and construction techniques are understood, the circuits can be applied. *Figure 3* shows a way to achieve very precise control of a rotary pump, used to feed a biochemical fermentation process. In this example, the exponentiator, composed of Q1 and A1A, is driven from input amplifier A1D. Q1B's collector current, instead of biasing a voltage output amplifier as in *Figure 2*, pulls current from the A1B integrator which ramps up (trace A, *Figure 4*) until it is reset by level triggered A1C (A1C output is trace B, *Figure 4*). The 100 pF capacitor provides AC positive feedback to A3C's "+" input (trace C, *Figure 4*). The magnitude of the current that Q1B's collector pulls from A1B's summing junction will set the frequency of operation of this oscillator. Note that the operation of the exponentiator is similar to the basic circuit in *Figure 3* because A1B's summing junction is always at virtual ground. A1C's output drives the MM74C76 flip-flop to bias the output transistors with 4-phase drive for a stepper motor which runs the pump head. In practice, the exponentiator allows very fine and predictable control for very slow pump rates (e.g., 0.1 rpm-10 rpm of the stepper motor), aiding tight feedback control of the fermentation process. When high pump rates are required, such as during process start-up or when a wide feedback control error exists, the exponentiator can be voltage directed to the top of its range. To calibrate the circuit, ground V_{IN} and adjust the 0.1 Hz trim until oscillation just ceases. Next, apply 7.5V at V_{IN} and adjust the 600 Hz trim for 600 Hz output frequency. *Figure 5* shows a circuit similar to *Figure 3*, except that a more accurate V-F converter is used. This circuit is intended for laboratory and audio studio applications requiring an oscillator whose frequency changes exponentially with an applied input sweep voltage. Applications include swept distortion measurements (where this circuit's output is used to drive a sine coded ROM-DAC combination or analog shaper) and music synthesizers. The V-F converter employed allows better than 0.15% total conformity over a range of 10 Hz-30 kHz. The voltage reference used to drive A1A's input resistor is derived from the LM331A's internal reference and is scaled by A1B, which also biases the zero trim setting. The DM74C74 provides a square wave output for applications requiring a waveform with substantial fundamental frequency content. The 0.15% conformity performance achieved by this circuit will meet almost any synthesizer or swept distortion measurement and the scale factor may be easily varied. To trim, apply OV to the input and adjust zero until oscillation (typically 2 Hz-3 Hz) just starts. Next, apply −8V and adjust the 5k unit for an output frequency of 30 kHz. For the values given, the K factor of the exponentiator will yield a precise doubling in frequency for each volt of input (e.g., 1V in per octave out).

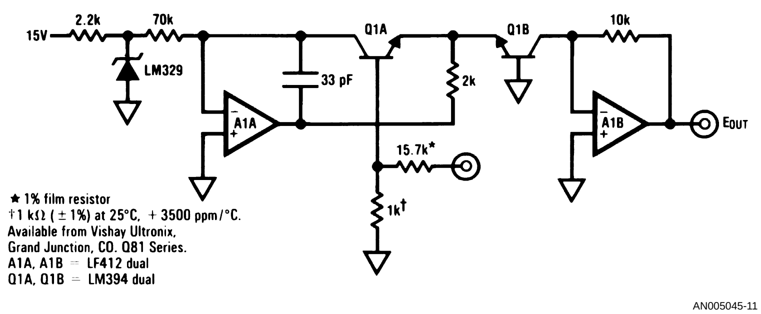


FIGURE 2.

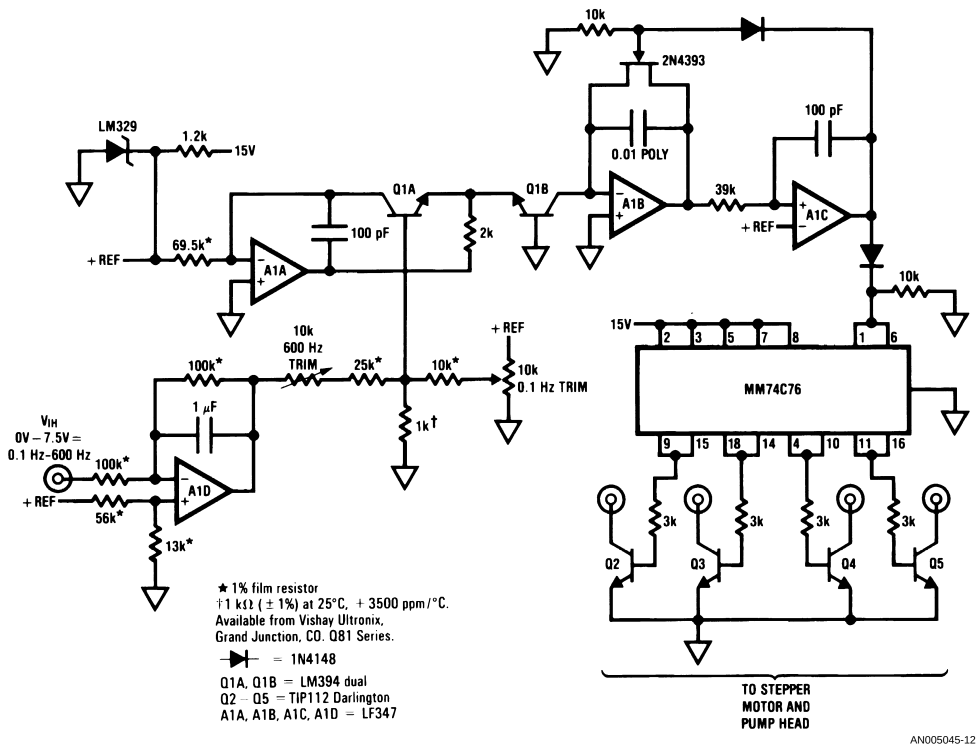


FIGURE 3.

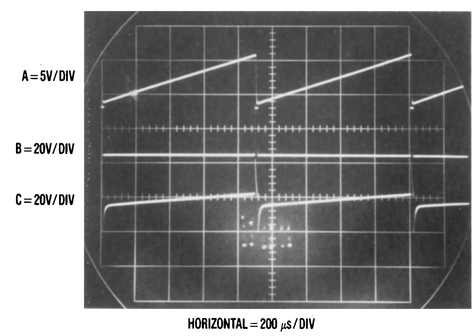
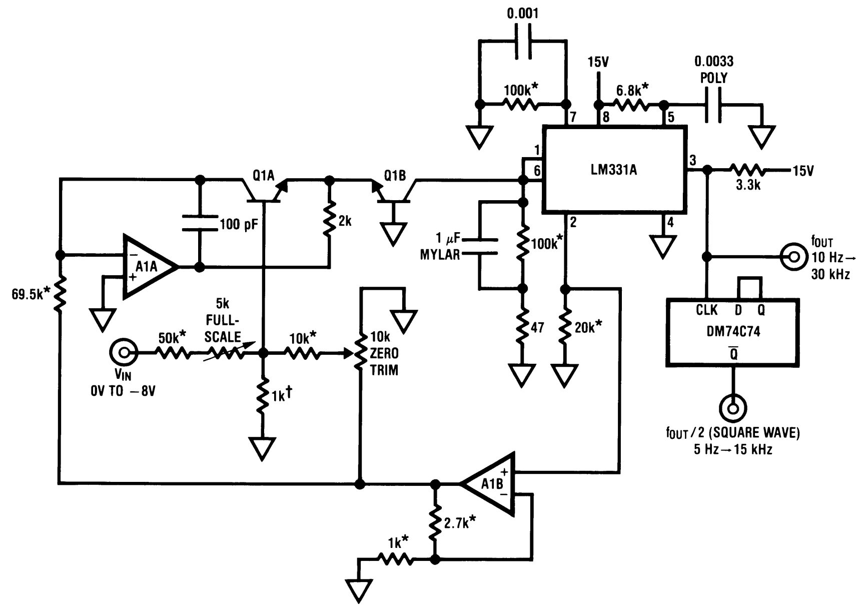


FIGURE 4.



*1% film resistor

†1 kΩ (±1%) at 25˚C, +3500 ppm/˚C.
Available from Vishay Ultronix,
Grand Junction, CO, Q81 Series.
A1A, A1B = LF412 dual
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FIGURE 5.

Figure 6 shows a way to use the exponentiator circuit in a non-invasive, high reliability gas gauge which was designed for use in irrigation pump arrangements in remote locations. The application calls for a highly reliable gas gauge to be retrofitted to large fuel tanks which supply pump motors. It is desirable to run the gas tanks down as closely to empty as possible to eliminate condensation build-up without running out of fuel. This acoustically-based scheme operates by bouncing an ultrasonic pulse off the liquid level surface and using the elapsed time to determine the fuel remaining. This time is converted to a voltage, which is exponentiated to provide a readout with high resolution for nearly empty tanks. The 60 Hz derived clock pulse (trace A, *Figure 7*) drives the transistor pair to bias the ultrasonic transducer with a 100V pulse. Concurrently, the DM74C74 flip-flop is set high (trace C, *Figure 7*) and the DM74C221 one-shot (trace D, *Figure 7*) is used to disable the output of the receiver amplifier. The acoustic pulse bounces off the gasoline's surface and returns to the transducer. By this time, the disable pulse has gone low and the A1A, A1B, A1C and C1 receiver responds (trace B, *Figure 7*) to the transducer's output. C1's output resets the flip-flop low via the DM74C04 inverter. The width of the 60 Hz flip-flop output pulse represents the transit time and the fuel remaining. This width is voltage clamped and integrated at A1D, whose output drives the exponentiator. The 1V/decade scale factor of the exponentiator means that the last 20% of the meter scale corresponds to a tank with only 2% fuel remaining. The first 10% of the meter indicates 80% of the tank's capacity.

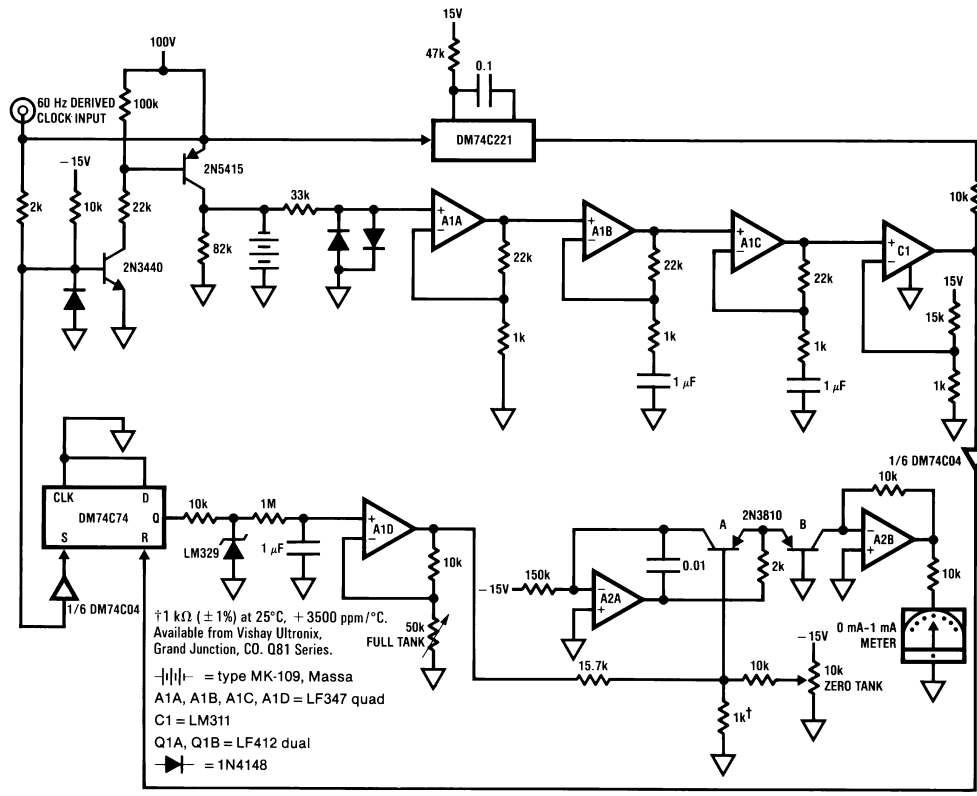

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FIGURE 6.

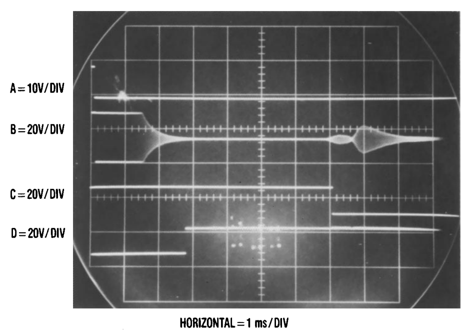

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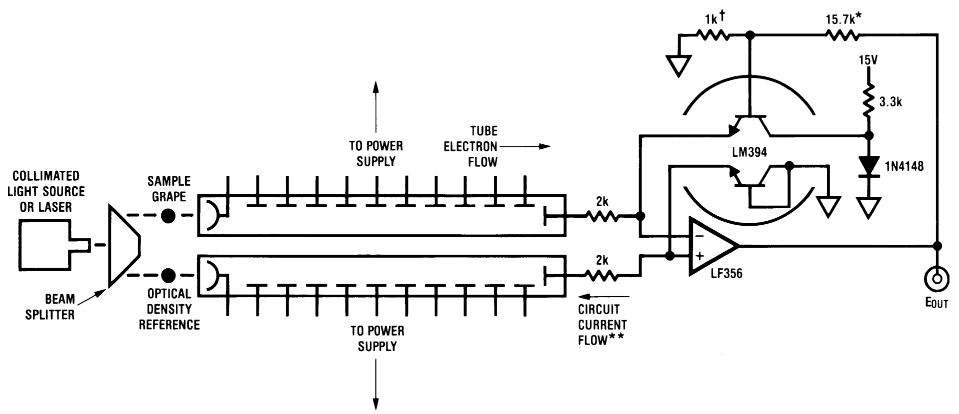
FIGURE 7.

The last application determines density by using photometry. In this arrangement, a light source is optically split (*Figure 8*) and the resultant two beams drive light through a sample and an optical density reference. In this case, the optical sample is a grape, and the photometric set-up is used to correlate the optical density of the grape with its ripeness. Two photomultiplier tubes detect the light passed by the sample and the reference. The ratio of the photomultiplier outputs, which may vary over a wide range, is dependent upon the optical density difference of the sample and the reference. The tubes' output feed a log *ratio* amplifier. This configuration dispenses with the fixed current reference normally employed, and substitutes the output of the reference channel photomultiplier. In this fashion, the log amplifier's output represents the ratio between the densities of the sample and reference channels over a wide dynamic range. Variations in the light source intensity have no effect. Strictly speaking, the LF356 inputs are not at virtual ground, and an imperfect current-to-voltage conversion should result. In fact, the output impedance of the photomultipliers is so high that errors are minimal. The most significant log conformance error source in this simple log circuit is the fact that the transistor's collectors are at slightly different potentials. For the application shown, this uncertainty is not significant.

REFERENCES

Non-Linear Circuits Handbook; Analog Devices, Inc.

Logarithmic Converters, Application Note AN-30;
R. C. Dobkin; National Semiconductor Corporation.



*1% film resistor
†1 kΩ (±1%) at 25˚C, +3500 ppm/˚C.
Available from Vishay Ultronix,
Grand Junction, CO, Q81 Series.
**≈ 10 µA nominal but may vary from
10^{-4}A to 10^{-9}A

FIGURE 8.

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